in our Curve, the greateſt Breadth is when the Point F *divides the Line* A B *in extream and mean Proportion: whereas in the* Foliate *it is when* A B *is triple in power to* B F. *And the greateſt* E F *or Ordinate in the* Foliate *is to that of our Curve nearly as* 3 *to* 4, *or exactly as* $\sqrt{\tfrac{2}{3}\sqrt{\tfrac{1}{3}}-\tfrac{1}{3}}$ *to* $\sqrt{5\sqrt{\tfrac{5}{4}}-5\tfrac{1}{2}}$.

But ſtill theſe Differences are not enough to make them two diſtinct Species, *they being both defined by a like Equation, if the* Aſymptote S G P *be taken for the Diameter. And they are both comprehended under the* fortieth Kind *of the* Curves *of the* third Order, *as they ſtand enumerated by Sir* Iſaac Newton, *in his incomparable Treatiſe on that Subject.*

IV. *An eaſy Mechanical Way to divide the Nautical Meridian Line in* Mercator's *Projection; with an Account of the Relation of the ſame Meridian Line to the* Curva Catenaria. *By* J. Perks, M. A.

THE moſt uſeful Projection of the Spheric Surface of Earth and Sea for Navigation, is that commonly call'd *Mercator*'s; tho' its true Nature and Conſtruction is ſaid to be firſt demonſtrated by our Countryman Mr. *Wright*, in his *Correction of the Errors in Navigation*. In this Projection the Meridians are all parallel Lines, not divided *equally*, as in the common plain Chart (which is therefore erroneous,) but the Minutes and Degrees (or ſtrictly, the *Fluxions of the Meridian,*) at every ſeveral Latitude are proportional to their reſpective *Secants*. Or a Degree in the projected Meridian at any Latitude, is to a Degree of Longitude in the Equator, as the *Secant* of the ſame Latitude is to *Radius*.

The

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The Reaſon of which Enlargement of the Elements of Latitude is, to counterbalance the Inlargement of the Degrees of Longitude. For in this Projection, the Meridians being all parallel, a Degree of Longitude at (ſuppoſe) 60 Deg. Lat. is become equal to a Degree in the Equator, whereas it really is (on the Globes Surface) but *half* as much, the Radius of the Parallel of 60 Deg (that is its *Coſine*) being but *half* the Radius of the Equator. Therefore to proportion the Degrees of Latitude to thoſe of Longitude, a Degree (or Elemental Particle) in the Meridian, is to be as much greater than a Degree (or like Particle) in the Equator, as the Radius of the Equator is greater than the Radius of the Parallel of Latitude, *viz.* its *Coſine.*

In *Fig.* 5 let the Radius CD repreſent half of the Equator, DM an Arc of the Meridian; MS its Sine, CE its Secant; then is CS equal to its *Coſine* : and $CS : CM :: CD (= CM) : CE$, that is, as Coſine : to Radius :: ſo is Radius : to Secant. The Coſines being then, in this Projection, ſuppos'd all equal to Radius, or (which comes to the ſame) the Parallels of Latitude being all made equal to the Equator, the Radius of the Globe, at every point of Latitude, (by the precedent Analogy) is ſuppoſed equal to the Secant of Latitude; and conſequently the Elements (Minutes, *&c.*) of the Meridian muſt be proportional to their reſpective Secants,

The Way Mr. *Wright* takes for making his Table of *Meridional Parts*, is by a continual Addition of Natural Secants, beginning at 1 Minute, and ſo proceeding to 89 Deg. Dr. *Wallis* (in *Phil. Tranſ.* No. 176.) finds the Meridional Part belonging to any Latitude by this *Series*, putting S for its Natural Sine, *viz.* $S + \frac{1}{3}S^3 + \frac{1}{5}S^5 + \frac{1}{7}S^7 + \frac{1}{9}S^9$ &c. which gives the *Merid. part* required: How to find the ſame Mechanically by means of an eaſily-conſtructed Curve Line, is what I ſhall now ſhew.

1. Prepare a Rular *A B* (*Fig.* 2.) of a convenient Length, in which let *B o* be equal to the Radius of the intended Projection. To the Point *o* as a Center (on the narrower Edge of the Rular) faſten a little Plate-Wheel *w h* tight to the Rular, and of a Diameter a little more than the thickneſs of the Rular. Let *K R* (*Fig.* 3.) repreſent another long Rular, to which *A R* is a perpendicular Line. Place the Rular *A B* upon the Line *A R*, with the Center of the Wheel at *A*. Then with one Hand holding faſt the Rular *K R*, with the other Hand ſlide the end *B* of the Rular *A B* by the Edge of *K R*; ſo will the little Wheel *w h* deſcribe on the Paper a Curve Line *A C B*, to be continued as far as is convenient.

2. Having drawn the Curve *A C B*, draw a ſtreight Line *K R* by the Edge of the Rular *K R*: which Line is the *Meridian* to be divided, and alſo an Aſymptote to the Curve *A C B*.

3. In this Meridian, (accounting *R* to be the Point of its Interſection with the Equator,) the Point anſwering to any Degree of Latitude is thus found. In the perpendicular *A R*, make *R G* equal to the *Coſine* of Latitude (Radius being *A R*,) and from *G* draw *G C* parallel to *K R*, and interſecting the Curve in *C*. With Center *C* and Radius $C M = A R$, ſtrike an Arc cutting the Meridian at *M*; ſo is *M* the Point deſir'd.

4. In the Curve *A C*, let *c* be a Point infinitely near to *C*, and *c m*, $(= C M,)$ a Tangent to the Curve at *c*, making the little Angle *M C m*, to which let the Angle *R A r* be equal: ſo is $R r = M d$ (a Perpendicular from *M* to *c m*.) Draw *C D* equal and parallel to *A R*, interſecting *K R* in *S*. With Center *C* and Radius *C D* draw the Arc *D M*, and its Tangent *D E* and Secant *C E*.

5. Becauſe of the like Triangles *C D E*, *M d m*; $C D : C E :: M d : M m$, that is, as Radius to Secant of the Arc *D M*, (whoſe Coſine is $C S = G R$,) : : ſo is *M d*

($= Rr$ a Degree or Particle of the Equator :) to Mm the Fluxion or correſpondent Particle of the Meridian Line RM. Whence, and from what is premiſed concerning the Nature of this Nautical Projection, 'tis evident that RM is the *meridional Part* anſwering to the Latitude whoſe Coſine is GR. Or thus ; With Center R and Radius AR deſcribe the Quadrant $A\varkappa\alpha$, in which let the Arc $A\varkappa$ be equal to the given Lat. From $\varkappa$ draw $\varkappa C$ parallel to KR, and interſecting the Curve in C, ſo is $C\varkappa$ the Meridional Part deſir'd being equal to RM, as is eaſy to ſhew.

As to the other Properties of this Curve, tis evident, from its Conſtruction, that its *Tangent* (as CM) is a *Conſtant Line* every where equal to AR; the Curve being generated by the Motion of the Wheel at the End of the Rular which is its Tangent. And from hence the Curve ACB may, for diſtinction, be call'd the *Equitangential Curve.*

7. The Fluxion of the Area $ARMC$ is the little Sector or Triangle MCd, which ſame is alſo the Fluxion of the Sector CDM: whence the Areas $ARMC$, CDM are equal, and the whole Area ACB &c, KMR being infinitely continued, is equal to the Quadrant $AR\alpha$:

8. To find the Radius of Curvature of any Particle, as Cc, from C draw an indefinite Line CT perpendicular to CM, (on the concave ſide of the Curve) and from c another Line perpendicular to cm, which Lines, (becauſe of the Inclination of CM to cm,) will ſomewhere meet as at T, making an Angle $CTc = MCm$. Theſe Angles being equal, their Radii are proportional to their Arcs: therefore, $Md : Cc :: MC : CT$. But $Cc = dm$ (becauſe of $CM = cm$) ſo that $Md : dm$ ($:: CD : DE$) $:: CM : CT$. But $CD = CM$, therefore $CT = DE =$ Tangent of the Arc DM.

9. So that ſuppoſing *ATt* a Curve Line in which are all the Centers of Curvature of the Particles of *ACB*, any point as *T* being found as before, the Length *AT* (by the nature of *Evolution of Curves,*) is every where equal to the *Tangent* of its correſpondent Circular Arc *DM*. The Point *T* is alſo found by making *MT* perpendicular to *RM*, and equal to the Secant *CE*: for ſo is the Angle *CMT*=*MCD*; and the Triangle *MCT* equal to the Triangle *CDE*.

10. Let *AHh* be an Equilater Hyperbola whoſe Semiaxis is *AR* and Center *R*. In the Meridian let *RP* be equal to the Tangent *DE*. Join *AP*, and draw *PH*=*AP* and parallel to *AR*. Compleat the Parallelogram *HNRP*, ſo will the Point *H* be in the Hyperbola, and its ordinate *HN* (=*RP*=*DE*=*CT*) be equal to the Curve *ATt*. From whence, and from *Prop.* 3. *Coroll.* 2. of Dr. *Gregory's Catenaria* (*Phil. Tranſ.* N. 231.) it appears that the Curve *ATt* is that call'd the *Catenaria* or *Funicularia*, viz. the Curve into whoſe Figure a *ſlack Cord or Chain* naturally diſpoſes its ſelf by the Gravity of its Particles.

" 11. Hence we have another Property of the *Catenaria*
" not hitherto taken notice of (that I know of,) *viz.* that
" ſuppoſing *AR* (= *a*, the conſtant Line in Dr. *Gregory*)
" equal to the *Radius* of the Nautical Projection, and
" *RN* the Secant of a given Latitude, then is *NT* the
" *Catenaria's* Ordinate at *N*, equal to *RM* the Meridio-
" nal Part anſwering to the Latitude whoſe Secant is
" *RN*.

12. That *TA* is the *Catenaria* is alſo demonſtrable from Dr. *Gregory's* firſt *Prop.* Let *Tu* be the the Fluxion of the Ordinate *NT*; and *tu* (=*Nn*) the Fluxion of the Axe *AN*. Then becauſe of like Triangles *TCM*, *Tut*, *CM* : *CT* (=*TA*) :: *Tu* : *ut*, that is, as *CM* a conſtant Line to *TA* the Curve :: ſo is the Fluxion of the Ordi-

Ordinate, to that of the Axe ($\dot{y} : \dot{x}$) according to *Prop.* 1, *Catenariæ.*

13. From the Premiſes the Conſtruction and ſeveral Properties of the *Catenaria* are eaſily deducible; one or two of which I'll ſet down.

1, The Area *ATMR* is equal to *AOPR* a Rectangle contained by Radius *AR* and *RP* the Tangent anſwering to Secant *HP* = *TM.* For becauſe of the like Triangles *CMm*, *CEe*; *CM* : *CE* :: *Mm* : *Ee*, that is, putting r, s, t, m for Radius, Secant, Tangent and Meridional part *RM.*) $r : s :: \dot{m} : \dot{t}$ whence $r\dot{t} = s\dot{m}$, and all the $r\dot{t}$ = all the $s\dot{m}$, that is *AOPR* = *ATMR*, which agrees with Dr. *Gregory*'s *Cor.* 5. of *Prop.* 7.

14. Suppoſing the former Conſtruction, let be added the Line *RH*, including the *Hyperbolic Sector ARH.* I ſay the ſame Sector is equal to half the Rectangle *ARMQ* contained by Radius *AR* and the Meridional Part *RM*, ($= \frac{1}{2} rm$), For the Sector *ARH* = Triangle *RNH* wanting the Semiſegment *ANH.* The Fluxion of the Triangle *RNH* is $\frac{s\dot{t} + t\dot{s}}{2}$. The Fluxion of *ANH* is $t\dot{s}$. So the Fluxion of the Sector *ARH* is $\frac{s\dot{t} + t\dot{s}}{2} - t\dot{s} = \frac{s\dot{t} - t\dot{s}}{2}$, 'Tis found before (*Sect.* 13.) that $r : s\ (s : \frac{ss}{r}) :: \dot{m} : \dot{t}$; whence $s\dot{t} = \frac{ss}{r}\dot{m}$. And becauſe of the like Triangles *CDE*, *Efe*, *CD* : *DE* :: *Ef* : *fe*. But *Ef* = *Mm* = $\dot{m}$, becauſe both *Ef* and *Mm* are to *Md* in the ſame Reaſon, *viz.* as s to r; therefore $r : t\ (t : \frac{tt}{r}) :: \dot{m} : \dot{s}$: whence $t\dot{s} = \frac{tt}{r}\dot{m}$, and $\frac{s\dot{t} - t\dot{s}}{2} =$

$\frac{ss-tt}{2r}\dot{m}=\frac{rr}{2r}\dot{m}=\frac{1}{2}r\dot{m}$, = the Fluxion of the Hyperbolic Sector *A R H*, whoſe flowing Quantity is therefore equal to $\frac{1}{2}rm=\frac{1}{2}$ *A R M*. *Q. E. D.*

15, This ſhews another Property of the *Catenaria*, viz. that it ſquares the Hyperbola; for *R M* is equal to *N T* the Ordinate of the *Catenaria*.

16. In *Fig.* 4. Let *A R* be Radius, *A C B* the Equitangential Curve; *M R N* its Aſymptote, in which let *M*, *N*, be any two Points equally diſtant from *R*. Upon *M* draw *M L* parallel to *A R* and equal to the *Difference* of the Secant and Tangent of that Latitude whoſe Meridional Part is *R M* (by § 3, 4.) Upon *N* draw *N O* parallel to *A R*, and equal to the *Summ* of the foreſaid Secant and Tangent. Do thus from as many Points in the Aſymptote as is convenient, and a Curve drawn equably through the Points *L* - - - *A* - - - *O*, &c. will be a *Logarithmic Curve*, whoſe *Subtangent* (being conſtant) is equal to Radius *A R*.

17. Let *n o* be an Ordinate infinitely near and parallal to *N O*. $Op=Nn$ the Fluxion of the Aſymptote; *O T* the Tangent, and *T N* the Subtangent to the Logarith. Curve in *O*. Then $op:pO::ON:NT$. But $ON=s+t$; therefore $op=\dot{s}+\dot{t}$. $pO=\dot{m}$ (the Fluxion of the Meridian or Aſymptote.) So the Analogy is $\dot{s}+\dot{t}:\dot{m}::s+t:NT$. By *Sect.* 13, 14, $\dot{s}:\dot{m}::t:r$: alſo, $\dot{t}:\dot{m}::s:r$. and thence $\dot{s}+\dot{t}:\dot{m}::t+s:r$. wherefore is *N T* (the Subtangent to *L A O*) equal to Radius *A R*, a conſtant Line, and conſequently the Curve *L A O* is the Logarithmic Curve, and its Subtangent known.

18. The ſame Demonſtration ſerves for *L M*, (any Ordinate on the other Side of *A R*) only changing the Sine + into —; and then it agrees with Mr. *James Gregory*'s *Prop*. 3. *pag*. 17. of his *Exercitations*, viz. *That*

the Nautical Meridian is a Scale of Logarithms of the Differences whereby the Secants of Latitude exceed their reſpective Tangents, Radius being Unity. So here RM is the Logarithm of ML, the Difference of the Secant and Tangent of the Latitude whoſe Meridional part is RM.

19. Suppoſing the precedent Conſtruction, if through any point C of the Curve ACB be drawn a right Line GCW parallel to MR, terminated with the Logarithmic Curve in W and the Radius AR in G: I ſay that the ſame right Line WG is equal to the intercepted part of the Curve Line AC.

20. Let wg be a Line infinitely near and parallel to WG, and terminated by the ſame Lines; and CS, $W\sigma$, perpendicular to the Meridian; CS interſecting wg in z, and $W\sigma$ in y. Let CM be a Tangent to AC in C; $W\tau$ a Tangent to AW in W; ſo is $CM = \sigma\tau$. Becauſe of like Triangles Czc, CSM; and Wyw, $W\sigma\tau$; $CS : CM :: Cz : Cc$: alſo $W\sigma : \sigma\tau :: Wy : yw$. But $W\sigma = CS$; $\sigma\tau = CM$; $Cz = Wy$; therefore is yw the Fluxion of GW, equal to Cc the Fluxion of the Curve AC: Conſequently $GW = AC$. *q. e. d.*

It may be noted that this Equitangential Curve gives the Quadrature of a Figure of Tangents ſtanding perpendicular on their Radius. In *Fig.* 3. let $A\gamma\Gamma$ be a Curve whoſe Ordinates as $g\gamma$, $G\Gamma$, are equal to the Tangents of their reſpective intercept Arcs Ak, $A\kappa$. Let ΓG be produced to touch the Curve AC in C: then is the Area $A\Gamma G$ equal to the Rectangle contained by Radius AR and GC the produced part of the Ordinate; or $A\Gamma G = AR \times GC$. The Demonſtration of which, and of the following *Section*, I for Brevity omit.

22. If we ſuppoſe the Figure ACB &c. KR (*Fig.* 3.) infinitely continued, to be turned about its Aſymptote RK as an Axe, the Solid ſo generated will be equal to

a

rectangled Cone whoſe Altitude is equal to $A R$. And its Curve Surface will be equal to half the Surface of a Globe whoſe Radius is $A R$. So that if the Curve be continued *both ways* infinitely (as its Nature requires) the whole Surface will be equal to that of a Globe of the ſame Radius $A R$.

The Deſcription of the Rular and Wheel, *Fig.* 2. is ſufficient for the Demonſtration of the Properties of the Curve: but in order to an actual Conſtruction for Uſe, I have added *Fig.* 5. where $A B$ is a braſs Rular; $w h$ the little Wheel, which muſt be made to move freely and tight upon its Axe (like a Watch-Wheel) the Axe being exactly perpendicular to the Edge of the Rular. s repreſents a little Screw-pin to ſet at ſeveral Diſtances for different Radii, and its under End is to ſlide by the Edge of the other fixt Rular. p is a Stud for convenient holding the Rular in its Motion.

Note, *Moſt of theſe Properties of this* Curve *by the Name of* la Tractrice, *are to be found in a* Memoire *of M.* Bomie *among thoſe of the* Royal Academy *of* Sciences *for the Year* 1712. *but not publiſh'd till* 1715: *Whereas this Paper of Mr.* Perks *was produced before the* Royal Society *in* May 1714, *as appears by their Journal.*

VI. *An Account of a Book entituled* Methodus Incrementorum, *Auctore* Brook Taylor, LL.D. & R.S. Secr. *By the Author.*

WHEN I apply'd my ſelf to conſider throughly the Nature of the Method of Fluxions, which has juſtly been the Occaſion of ſo much Glory to its great Inventor Sir *Iſaac Newton* our moſt worthy Preſident, I fell by degrees into the Method of Increments, which I have endeavour'd to explain in this Treatiſe. For it being the Foundation of the Method of Fluxions that the Fluxions

Philoſoph. Tranſac. N°. 345.